A Critical and Historical Review of Various Experimental Methods for Determining the Distance to the Heliospheric Termination Shock.

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ABSTRACT

The solar wind, which consists of ionized atoms, streams continuously from the solar corona radially outward, carrying with it the solar magnetic flux through interplanetary space called the heliosphere. The solar wind is initially supersonic in the interplanetary medium, with a velocity between 350-450 km/s. When the solar wind velocity falls below approximately 200 km/s it becomes subsonic in the interplanetary medium. During the transition from the supersonic to subsonic regime the solar magnetic fields become compressed with the formation of a shock. This shock, called the solar wind termination shock, will accelerate charged particles. In this thesis I will compare and contrast different methods of analyzing measurements made in the heliosphere by instruments aboard various spacecraft in the last fifty years. Space-borne measurements not only suggest that the termination shock does in fact exist, but also allow one to calculate an approximate distance to this structure. One such method of calculating the distance to the termination shock entails the measurement of radio waves generated by particle acceleration at the shock. The other methods are primarily concerned with particle distributions in the heliosphere, specifically anomalous particles, which are singly charged ions accelerated in local interplanetary space. Finally, I will suggest possible improvements and new measurements that may be made in the future, including the direct measurement of the structure itself by the Voyager spacecraft.

I. Introduction

The bubble around our solar system that is dominated by the solar wind is the heliosphere. The solar wind is a plasma of ionized atoms that stream radially outward from the Sun's corona carrying with it the solar magnetic flux, whose field lines take on the form of an Archimedian spiral near the equator [1]. The polarity of the solar magnetic field is organized into two hemispheres separated by a thin neutral current sheet at the equator across which the field's polarity reverses directions throughout the 22-year solar cycle. The solar wind has a density near earth of $\sim 8*10^6$ protons/m^3; the density decreases as $1/r^2$ as it propagates outward. The initial velocity of the solar wind is 350-400 km/s, which is supersonic in the interplanetary medium. At some point the solar wind velocity must decrease to, at most ~25km/s – the known flow velocity of the nearby local interstellar plasma. When the solar wind velocity drops below supersonic speeds to a subsonic velocity of ~200 km/s the discontinuity in the flow velocity produces a discontinuous magnetic field. When this occurs, the solar magnetic fields perpendicular to the flow are compressed, and a shock is formed which accelerates charged particles to several hundreds of MeV. This structure is called the termination shock. The existence of the termination shock is supported by measurements made by various space-borne experiments and is the most probable explanation for the measured anomalous component of the cosmic rays [2]. Yet, the theoretical explanations of the experimental measurements are dependent upon various assumptions, made necessary due to the immense size and complexity of the heliosphere. The assumptions made as to the relative importance of particular heliospheric parameters have caused a wide range of predictions to be made over the years as to the distance of the heliospheric termination shock from the Sun. In this thesis paper I will compare and contrast different methods and instrumentation used to calculate the distance to the termination shock, with a review of the various research that has gone into understanding heliospheric phenomena from the 1950's to the present.

II. Heliospheric Models

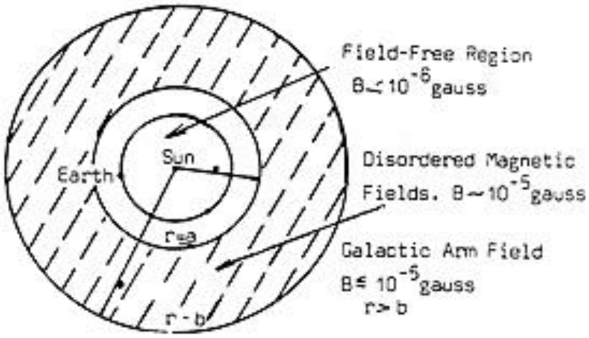
Davis made the first description of the heliosphere close to our current model in 1955 and presented his ideas at the 4th International Cosmic Ray Conference in Guanajuato, Mexico. [3, 4] Davis suggested that what we now call with some modifications the solar wind, created a cavity in the galactic magnetic field that allowed energetic solar flare particles to reach the earth. These particles were in fact measured indirectly with ground based neutron monitor instruments by Meyer Parker and Simpson (MPS) in 1956, giving credence to his theory [5]. The gyro-radius a, of a particle with energy 1 GeV, in a field of $B \sim 10^{-5}$ Gauss (the assumed value of the galactic, and at the time local magnetic field) would be on the order of a = .022 AU, ($1.5*10^{13}$ cm = 1A.U. Astronomical Unit, the distance between the earth and the sun) because:

$$cp_{\perp} = eBa$$

where c is the speed of light, $p_{\perp}$ is the momentum of a particle perpendicular to the magnetic field, and e is the proton's charge.

Since particles with energy of 1GeV were measured on the earth at 1AU by MPS, and a particle with a < 1AU could never intersect with the earth, Davis incorrectly surmised that there must be a field free cavity in the interplanetary medium. So the first simple model of the heliosphere was born. Davis' model is illustrated in figure 1:

Fig. 1



To estimate the size of the cavity, Davis equated the solar wind pressure, to the interstellar magnetic pressure and solved for r:

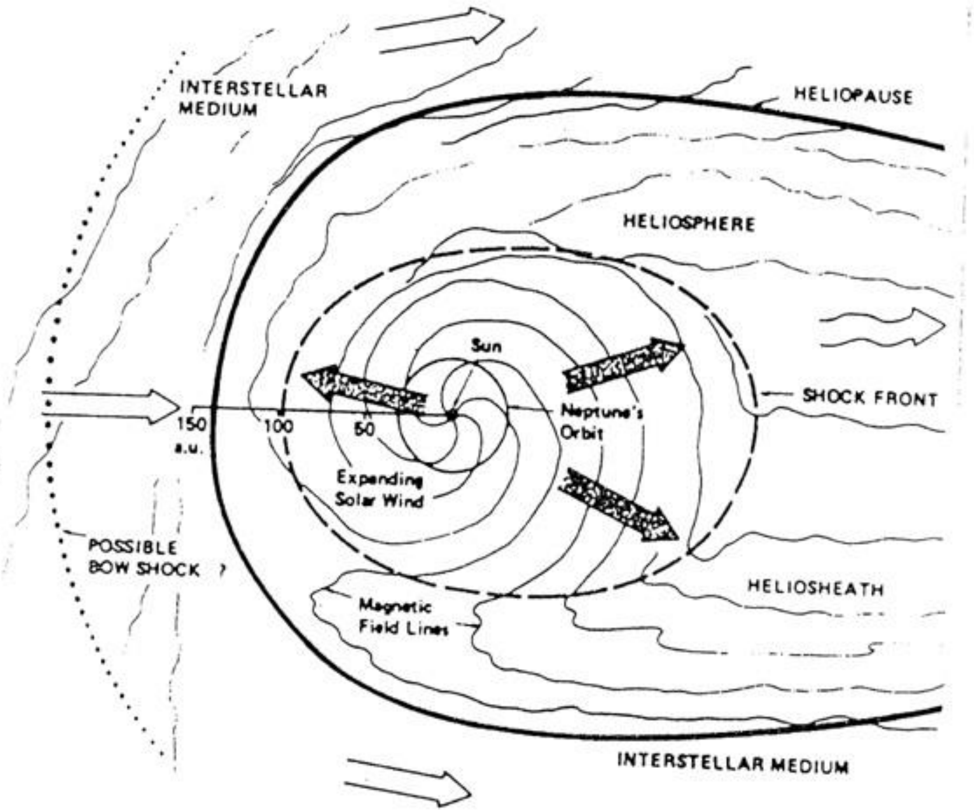
$$(1) \qquad n_p v_p^2 m_p (r_e/r)^2 = B^2/8\pi \qquad [6, 7]$$

where r_e = 1A.U. is the radial distance of the earth to the Sun, n_p is the proton number density at 1 A.U., v_p is their mean radial velocity, and m_p is the proton mass. The interstellar magnetic field B was taken to be the order of 10^{-5} Gauss (as previously mentioned) from various observations such as Faraday rotation of polarized extragalactic radio sources [8]. Due to an incorrect estimate of n_p, derived by Biermann at that time, Davis first overestimated the radial distance to the heliospheric boundary, the heliopause, to be ~2000 A.U. Davis later refined this value to r = 200 A.U. [6,7].

Davis' field free model though was incorrect. In actuality he underestimated the complexity of the heliosphere's magnetic structure. Davis had no reason to believe the sun had a magnetic field at all. Also, Davis' assumption that solar wind particles only had perpendicular momentum was incorrect, although he later made corrections to this assumption. It was not until the 1960's that a more sophisticated model of the heliosphere appeared.

The canonical basis for all heliospheric and solar wind models that qualitatively and quantitatively expand on Davis' cavity model was described first by Parker in 1961 and in more detail in 1963 with Parker's *Interplanetary Dynamical Processess*. [9, 1] Parker's hydrodynamic radially expanding solar wind model assumes that the quiet-day solar wind will flow outward past 1 A.U. with essentially constant velocity on the order of 300-400km/s with a mean density of 10-20 atoms/cm^3 at earth's orbit. Naturally, the solar winds density would fall off by $1/r^2$. In the Parker model, the solar wind will continue flowing outward at supersonic speed from the solar corona until its energy density, or pressure, decreases to that of the surrounding interstellar medium and the solar wind is diverted by the interstellar gasses and magnetic fields. One can see in figure 2 that Parkers model of the heliosphere expanded greatly upon the Davis model.

Fig. 2



Parker stated that the interstellar pressure p_i is composed of the hydrostatic pressure of the interstellar gas p_g, the hydrostatic pressure of the cosmic ray gas, p_c, and the interstellar magnetic field pressure p_b. [1] Parker calculated p_i to lie in the range of $1\text{-}4*10^{-12}$ dyne/cm^2. Building on Davis' model, mostly with his better understanding of the solar wind, Parker claimed that the heliopause occurs at a distance from the Sun on the order of r_d by equilibrating pressures, such that:

$$(2) \quad r_d=r_e((1/2)N_e m v^2/p_i)^{1/2} \qquad \text{with } N=N_e(r_e/r)^2$$

where N_e is the solar wind density at earth orbit, $r=r_e$. Parker's first calculated distance to the heliopause ranged from 45-90 A.U.

III. The Heliospheric Termination Shock

Clauser and Weymann first pointed out that the solar wind would take part in a shock transition to subsonic flow in the vicinity of the heliopause when the solar wind first begins to be influenced by the pressure of the interstellar medium [10, 11]. At the termination shock the temperature is low, so the Mach number M of the transition is large. The Mach number is the ratio of the relative speeds of a fluid and a rigid body to the speed of sound in that fluid under identical temperature and pressure. For $M > 1$ the fluid or body is moving at supersonic speed, likewise $M < 1$ is subsonic.

Parker showed how the solar wind velocities just ahead, and just behind the shock transition are related by the Rankine-Hugoniot relations for high Mach number, which relate the fluid velocity, density, and pressure, behind and ahead of the shock transition, and reduce to:

$$(3) \qquad U_2 = U_1((\Delta-1)/(\Delta+1)) \qquad N_2 = N_1((\Delta-1)/(\Delta+1)) \qquad p_2=2/(\Delta+1)$$

where in an ideal flow, $\Delta \cong 5/3$ is the ratio of specific heats for the transition. U is the solar wind velocity, N its number density and p its pressure; the subscripts 1 and 2 respectively refer to the upstream and down stream regions of the shock. If β is the ratio of specific heats beyond the shock transition, then every streamline in the flow in the subsonic regime is described by the Bernoulli equation. Using (2) (3) and the Bernoulli equation:

$$(4) \qquad (1/2)U^2+(\beta/(\beta-1))*(p_2/N_2m)= \beta/(\beta-1)*(p_2/N_2m)(\delta/p_2)^{(\beta-1)/\beta} \qquad \delta = \text{stagnation pressure}$$

A distance for the termination shock R_{ts} from the Sun may now be calculated:

$$(5) \qquad Rt_s = r_e^2(N_e m u_1^2/\delta)*(2/(\Delta+1))[1+((\Delta-1)(\beta-1))/4\beta)]^{\beta/(\beta-1)} \cong r_d$$

According to these calculations the distance to the termination shock can be shown to be approximately within the vicinity of the heliopause. Assuming a stationary termination shock, this would put a distance to the structure on the order of 45-90 A.U.

J.R. Jokipii first studied the possible acceleration of cosmic rays at the termination shock of the solar wind in detail in 1968 [11]. Jokipii studied the acceleration of fast charged particles by outward-moving magnetic irregularities at the solar wind boundary. He did this in order to explain observations during the same time period of low-energy galactic cosmic rays that suggested more than one source for these particles. This was due to the fact that current theories did not account for the observed energy intensities throughout the Galaxy. If cosmic ray particles were confined to the solar system long enough for them to receive energy from a source such as the termination shock, then acceleration by the solar wind interacting with the outer heliosphere could be considered as a new contributing factor to the local cosmic ray intensity.

The Lorentz force determines the basic physics behind particle acceleration at a shock front such as the termination shock:

$$\mathbf{F} = Ze(\mathbf{v}\mathrm{x}\mathbf{B}/c +\mathbf{E}) = d\mathbf{p}/dt; \quad \mathbf{E} = -\mathbf{V}\mathrm{x}\mathbf{B}/c$$

where **v** is the particle velocity, **p** is its momentum, and **V,** is the velocity of the plasma upstream to the shock or $\mathbf{V}_1$. The local magnetic and electric fields determine the magnitudes of **B** and **E**. Integration of this equation in specified fields will show the energy and position changes of particles interacting with the shock. At perpendicular or quasi-perpendicular shocks such as the termination shock, most of the acceleration is due to shock drift acceleration as the particles drift parallel to **E** while they interact with the shock. In this case, the magnetic fields in the plasma (magnetic fields embedded in the solar wind) that contribute most are those that are perpendicular to the shock front. In shock drift acceleration, particles increase their energy by an amount $Ze\mathbf{E}\bullet\Delta\mathbf{r}$, where $\Delta\mathbf{r}$ is the distance that the particles drift. Although this is the primary mode of acceleration at the termination shock, (contribution due to the $Ze\mathbf{E}$ term in the Lorentz force equation) a closed form solution for the energy changes of particles interacting with a shock dominated by this process does not exist. But, for the approximation where $\mathbf{V}_1\mathrm{x}\mathbf{B}_1 = 0$, where the directions of $\mathbf{V}_1$ and $\mathbf{B}_1$ are close to parallel, a closed form does exist (contribution due to the $\mathbf{v}\mathrm{x}\mathbf{B}$ term in the Lorentz force equation dominates). This solution is called first order Fermi acceleration. Since the solar magnetic fields frozen into the solar wind are in an Archimedean spiral, and are mostly parallel to the shock front in an ideal scenario, shock drift acceleration is dominant over Fermi acceleration at the termination shock.

Fig. 3

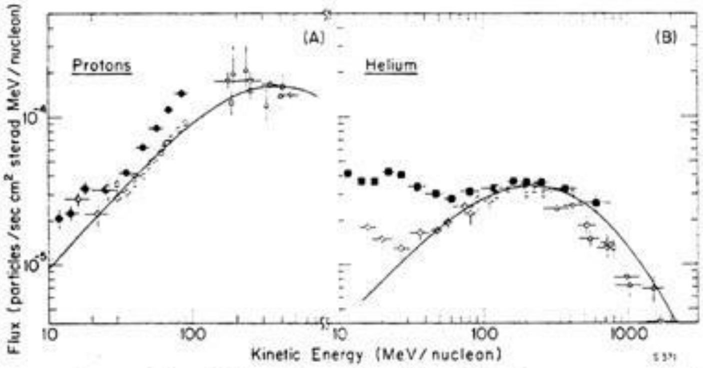
$\psi_1 = 90^\circ$ corresponds to the maximum energy gain from shock drift acceleration. ($\mathbf{E} = \mathbf{V}_1\mathrm{x}\mathbf{B}_1$ term)
$\psi_1 = 0^\circ$ corresponds to the maximum energy gain from first order Fermi acceleration. ($\mathbf{v}_1\mathrm{x}\mathbf{B}_1$ term)

IV. The Anomalous Component of the Cosmic Rays

Although the heliospheric termination shock theorized by Clauser and Weymann is theoretically possible in the context of Parker's heliospheric model, its existence is not required. For instance, if the transition from supersonic to subsonic flow occurs less rapidly than $1/r^2$, over a distance greater than ~1A.U., a particle accelerating shock front will not necessarily be formed. The most convincing evidence that the termination shock does in fact exist is the anomalous component of the cosmic rays.

The anomalous helium component was first observed in the quiet time cosmic ray flux in 1973 by Garcia-Munoz, Mason, and Simpson (GMS) using a charged particle detector on board the University of Chicago IMP-5 satellite experiment [12]. The term anomalous has been applied to particular constituents of measured charged particles because their spectral forms could not be explained by the standard spherically symmetric modulation theory for any choice of parameters that adequately replicate the spectra of other non-anomalous cosmic ray components. Soon after GMS made their measurements, Hovestadt, Vollmer, Gloeckler and Fan (HVGF) measured an increased oxygen flux between 2 and 8.5 MeV/nucleon using their ultralow energy telescope (ULET) on board the IMP-7 satellite [13]. In 1974 Mcdonald, Teegarden, Trainor, and Webber (MTTW) measured a new component of Nitrogen and Oxygen with the Goddard-University of New Hampshire cosmic ray telescope on the Pioneer 10 spacecraft [14]. It was discovered that the fluxes of helium, nitrogen, oxygen, and neon were observed to be enhanced in a region of the energy spectrum ranging from a kinetic energy of 20 MeV up to possibly 300 MeV. Considering a lack of similarity to solar particle event compositions, but more importantly, that an increasing radial density gradient was also observed by spacecraft propagating away from the center of the solar system, it was suggested that the origin for the anomalous component of the cosmic rays existed somewhere outside the heliosphere. Figure 4 demonstrates the anomalous component's divergence from the known energy spectrum of the galactic cosmic rays:

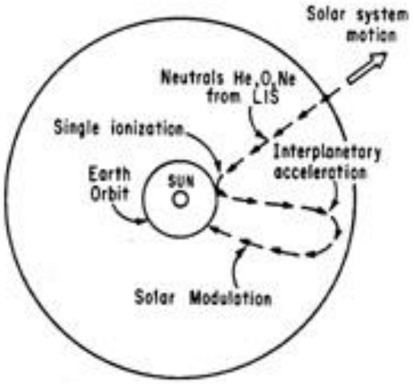
Fig. 4



Quiet-time spectra of protons (*a*) and helium (*b*) near solar minima. ***Solid points***, measurements by GMS (1977*a*) in 1976. ***Open points***, measurements in 1965 compiled by Gloeckler and Jokipii (1967). ***Solid lines***, predicted spectra for interstellar spectra of the form used by GMS (1975*c*) and a radial diffusion coefficient $\kappa = 110\,\beta P \exp[(r-1)/33]$ cm^2 s^{-1} with r in AU, P in GV.

In 1974 Fisk, Kozlovsky, and Ramaty (FKR) gave the first clear explanation of the origin of the anomalous component of the cosmic rays [2]. FKR proposed that particles of high first ionization potential (such as H, He, N, O, and Ne) originating in neutral interstellar gas clouds could penetrate the solar cavity when overtaken by the motion of the Sun through the interstellar medium. Once inside the solar cavity FKR predicted that some of the penetrating neutral atoms would be ionized by photo ionization from solar ultraviolet radiation or by charge exchange with solar-wind particles. Upon ionization these particles would be carried outward with the solar wind, and then accelerated to energies of about 1 keV per nucleon by some unknown accelerating mechanism. These particles would differ from solar wind particles by the fact that they are singly ionized and have energies considerably higher than solar wind thermal energies. FKR found that their mechanism could enhance N and O at ~10 MeV per nucleon over other elements in agreement with the observations of HVGF and MTTW. Figure 5 illustrates the FKR model.

Fig. 5



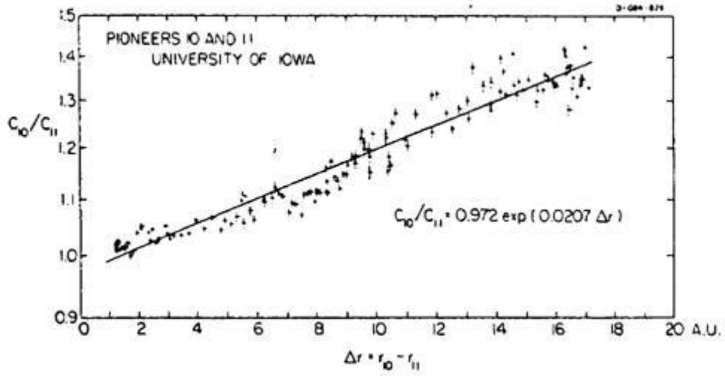
Pesses, Jokipii and Eichler (PJE) demonstrated in 1981 that the most likely acceleration mechanism in the FKR model was the heliospheric termination shock, by the method of diffusive shock acceleration, which they had explored in detail. In their model the intensity would increase with the positive radial gradient in agreement with observations [15]. Also the PJE model predicted an anomalous component intensity dependence on the 22-year solar magnetic cycle that would explain observed but previously unexplained intensity fluctuations in the anomalous component between 1965 and 1979. These intensity fluctuations led McKibben et al to suggest a dependence of the anomalous component on the sign of the Sun's magnetic field which reverses its sign in successive 11 year sunspot cycles [16, 17, 18]. PJE also observed that acceleration of singly ionized ex-interstellar particles such as the anomalous component would be favored for acceleration by a shock such as the termination shock. Previous attempts at modeling the anomalous component did not account for these features.

IV. Calculating the Radial Distance to the Termination Shock

A. Radial Gradients of Cosmic Rays

By examining radial gradients of cosmic rays it is possible to estimate a distance to the termination shock. Average radial gradients of cosmic rays in the outer heliosphere are generally computed from the logarithmic ratio of particle fluxes at two spacecraft divided by their separation distance. One of the first attempts at calculating a distance to the modulation boundary in this manner was made by Randall and Van Allen (RVA) in 1986 using cosmic ray intensity data (Energy T > 80 MeV/ nucleon) over a complete cycle of solar activity from instruments on board Pioneers 10 and 11 which were at a distance of 38AU and 21 AU respectively during the time of publication in 1986 [19a]. Included is a reproduction of RVA's measured radial gradient from detectors on board Pioneers 10 and 11 from the period 1972-1985 [19a and 19b]:

Fig. 6



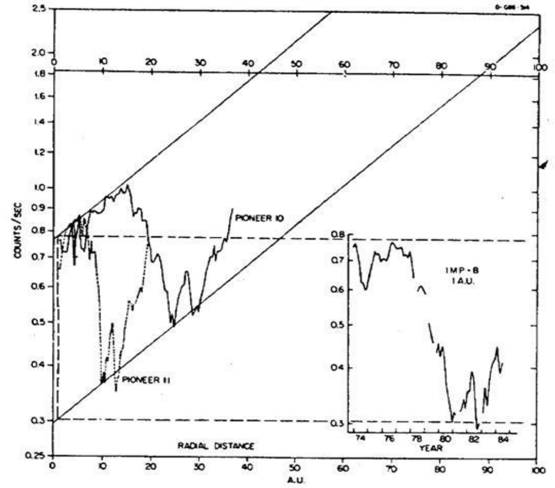
A sample plot from Van Allen and Randall [1985] illustrating our determination of the mean radial gradient of cosmic ray intensity using data from Pioneers 10 and 11.

A straight line connecting two corresponding points on the Pioneer 10 and 11 curves establishes the apparent mean radial gradient of intensity. RVA assumed cylindrical symmetry around the polar axis of the ecliptic plane and determined an observed integral radial gradient of $G = .0207\ (\mathrm{AU})^{-1}$. RVA assumed that the radius R of the modulation region is given by:

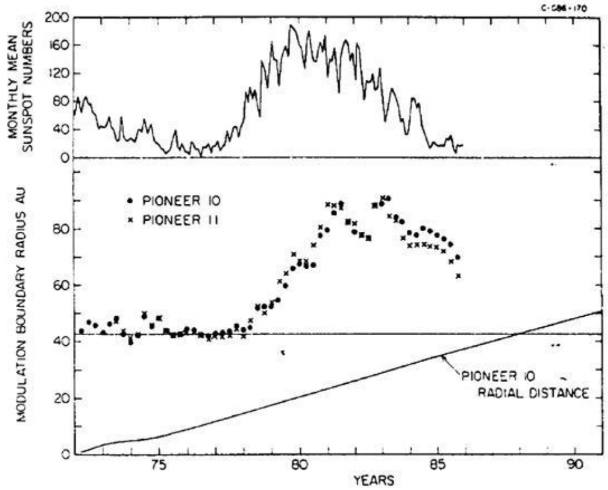
$$R = r - (1/G)\ln(C(r)/C(R))$$

where C(r) is the observed counting rates as a function of time and distance and C(R) is an theoretically derived value for the interstellar counting rate of 1.83 counts/sec assuming the formalism of Gleeson and Axford's demodulation model (20). See Appendix A.

Fig. 7 and 8 reprinted from RVA:



The two principal curves give the counting rates of detectors C on Pioneers 10 and 11 as a function of radial distance, with the phase of the solar activity cycle as the implicit variable. The two slanting straight lines that bound these curves are drawn for a radial gradient of 0.0207 $(AU)^{-1}$. The horizontal line near the top of the figure at 1.83 count/sec represents our inferred value of the interstellar counting rate. In the lower right of the figure is a plot of the time-dependence of the cosmic ray intensity as measured by IMP-8 at 1 AU [courtesy of S. M. Krimigis and R. B. Decker].



The central portion of the figure shows inferred values of the heliocentric radius of the cosmic ray modulation boundary in astronomical units (AU) over the recent solar activity cycle. The upper portion gives a plot of monthly mean sunspot numbers for the same period. The smooth curve in the lower portion gives the heliocentric radial distance of Pioneer 10.

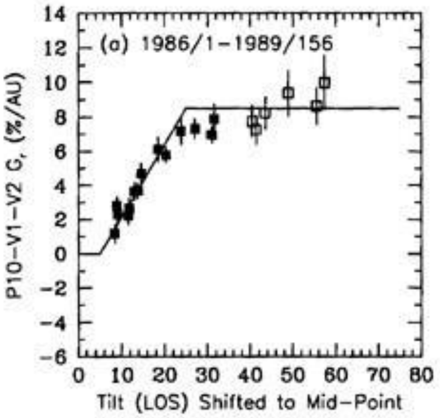
As can be seen in figure 7 and 8, Randall and Van Allen took into account the motion of the termination shock due to the changing solar wind intensity, and came up with a range of radial distances to the modulation boundary. The modulation boundary can be assumed to begin at the termination shock. RVA predicted that the distance to the termination shock is between 42 AU at the time of minimum solar activity in 1976, and 88AU about 1.5 years following the time of maximum solar activity in 1980. The changes in the termination shock distance are the result of changes in the dynamic pressure of the solar wind throughout the solar cycle.

Throughout the 1990's Stone, Cummings and Webber (SCW), have used the radial intensity gradients of the anomalous component of the cosmic rays measured on the Voyager and Pioneer spacecraft, along with values for the tilt of the heliospheric neutral current sheet, to explain the time variation of the radial gradient of the anomalous component of the cosmic rays. They then used the gradients and their relationship to the neutral current sheet to infer the location of the solar wind termination shock [23-29]. As more data has been accrued by space bourne charged particle instruments over the years, SCW have calculated values from 62 to ≤ 92 AU. I will concentrate here on their most recent results published by Stone and Cummings in 1999 at the 26th International Cosmic Ray Conference in Salt Lake City, Utah [29].

Theoretical and observational evidence suggests that during the ~11 year period when the polar magnetic field of the Sun's northern hemisphere is directed inward, the positively-charged particles are thought to enter the heliosphere by drifting along the heliospheric neutral current sheet (HCS) when its degree of tilt is $\leq 30^{\circ}$. In an outwardly directed polar magnetic field charged particles radially diffuse inward at higher latitudes and drift outward along the current sheet. This, leads to a different relationship between the radial gradient at low latitudes and the tilt of the current sheet [30-31]. SCW conclude that the degree of tilt of the current sheet may be expected to affect the access of the anomalous cosmic rays resulting in a relationship between the current sheet tilt and the intensity gradient. Reducing the tilt of the neutral current sheet decreases the distance that particles must traverse in an inward drift and, therefore, increases the observed intensity. The heliospheric current sheet can be visualized as frozen into the expanding solar wind, so that at any given time the tilt values extend from the Sun all the way to the termination shock in a series of steps proportional to the solar wind speed. Thus, the radial profile of the tilt translates into a radial profile of the gradient. With an assumed anomalous cosmic ray source intensity and shock location, the anomalous cosmic ray intensity at any position interior to the shock during any time period may be calculated.

The time history of the tilt angle is available from solar magnetic field observations by the Wilcox Solar observatory for each 27.3 day Carrington rotation from 1976 to the present. The following figure from Stone and Cummings 1999 paper shows measurements of anomalous oxygen from instruments on the Voyager and Pioneer spacecraft around the 1987 solar minimum as a function of tilt angle for a negative polarity (directed inward) field at an average radial location of 33 AU.

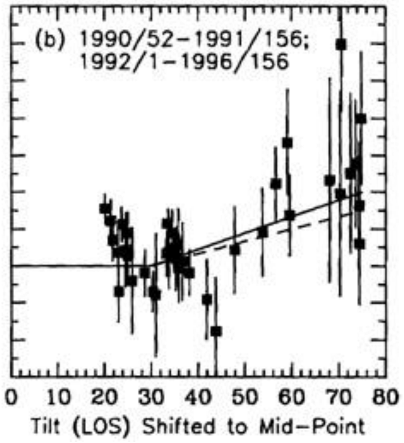
Fig. 9



It is apparent in figure 9 that at tilt values less than ~25° the gradient made between the radial positions of Voyager II and Pioneer 10 is remarkably proportional to the heliospheric current sheet tilt. Above 25° the gradient is no longer sensitive to the tilt, and may be due to radial diffusion which allows the particles to "short circuit" the large amplitude waves of the heliospheric current sheet at the large tilt angles [32]. The solid line is the Gr(tilt) relationship used in later calculations by Stone and Cummings.

Figure 9 illustrates measured data for positive polarity, at an average radial location of ~50AU during solar maximum. The figure presents measurements of anomalous oxygen between Voyager and Pioneer spacecraft as a function of tilt angle.

Fig. 10



A variation of gradients at the higher tilt angles can be seen in figure 10. This is due to the increased levels of transient perturbations in the "wavy" current sheet that occur at solar maximum. The gradient vs. tilt relationship is also different for the positive polarity field as expected. The gradients are very small when the tilt is ≤ 35°. Particle drifts insure that the radial gradients at lower latitudes reflect the much smaller radial gradients expected at higher latitudes.

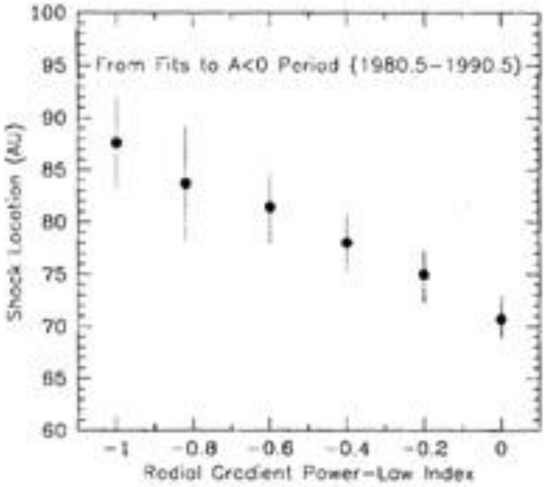
In order to calculate the distance to the termination shock Stone and Cummings used the lines in figures 8 and 9 in order to interpolate the gradient vs. tilt value relationship for any tilt value. The assumed form of the relationship being

$$Gr = Gr(tilt)(r/r_{obs})^n.$$

Stone and Cummings allowed n to vary to best-fit observations from 1980.5 to 1990.5 near the heliographic equatorial plane. As the heliosphere transitions from negative to positive polarity the maximum anomalous component intensity at the shock is expected to move from the equatorial region of the heliosphere to the heliospheric poles. Therefore, an exponential increase with time prior to the maximum intensity in1987, followed by an exponential decrease thereafter, restricts the shock intensity at the helioequator. The solar wind speed is also fixed at 440 km/s forcing each Carrington tilt observation to correspond to 6.93AU in radial range.

Cummings and Stone found their model to reproduce observations in the heliosphere for radial distances > 11.4 AU, and with fixed gradients, up to 1AU. For 1980.5 to 1990.5, the best fit termination shock locations as a function of the power law index n gave values for the termination shock distance between 69AU and 92AU with a best fit value of 83.7 AU for n = -.082. Figure 11 reproduced from Stone and Cummings 1999 paper shows the radius of the termination shock as a function of the radial gradient power law index:

Fig. 11



Lopate and Simpson (LS) also have made estimates of the radial distance to the termination shock [33]. LS used measurements of the anomalous component fluxes of helium and oxygen, obtained with the University of Chicago's charged particle instruments on board Pioneer 10 at 56 AU, and the earth orbiting satellite IMP 8 at 1AU. The charged particle telescopes are pulse-height analyzed and a dE/dx vs. E. technique is used to identify individual nucleonic species. Assuming that the energy density of the anomalous components will dominate the galactic cosmic ray and local interstellar magnetic field energy density at the termination shock, the energy density of the solar wind will then be nearly equal to that of the anomalous component at the site of the termination shock. Like previous attempts at estimating the radial distance of the termination shock, it is necessary to measure the energy densities and the flux levels of the anomalous components as a function of radial distance from the sun. LS used a standard 2 dimensional modulation model determined by the Fokker-Planck diffusion equation, which models the complex propagation of charged particles in the heliosphere, and their interactions with the chaotic heliospheric magnetic fields. Charged particles in the heliosphere are subject to three important influences modeled by the diffusion equation:

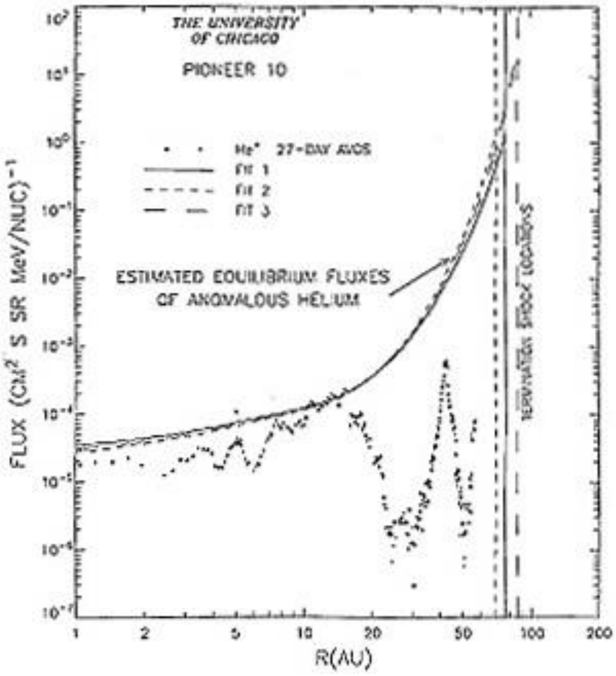
- Diffusion - the random walk of particles in the heliosphere
- Convection and particle drift - the tendency of charged particles to move outward with the solar winds magnetic fields
- Adiabatic deceleration - a particle cools when it moves through an expanding gas at constant temperature

See Appendix B for the details of this monster of an equation.

The diffusion equation can be numerically solved for an energy spectrum U(R,E) with the boundary condition $U_o(R_o,E_o)$, for $R \leq R_o$, where R_o is the distance to the source (the termination shock) and E_o is the energy of the source particles at R_o. The spectrum for each of the accelerated particles becomes a power law at high energy, which can be integrated to get energy density. In this particular case LS assumed the termination shock to be a strong shock of compression ratio 4, (ratio of the mean upstream plasma pressure to the mean downstream plasma pressure) which gives an energy spectrum $U \propto E^{-2}$. A value for the distance to the termination shock, R_o, can be calculated in a "backwards" fashion. First, one must choose various different values for R_o. Then, by substituting R_o into the diffusion equation, one can numerically determine energy spectra at different positions inside the heliosphere, $R<R_o$, where spacecraft measurements have been taken. By comparing the data accrued by the spacecraft to model spectra, for each given R_o, and using a χ^2 fit, the "best fit" R_o can be determined.

LS assume that the anomalous component was injected into the shock as a monoenergetic beam of particles with energy 10 keV and that the magnetic field strength of the shock is 5μG. Using this method LS calculated values of R_o between 70 and 90 AU. Figure 12 shows fits for different values of diffusion coefficients in the diffusion equation, and a maximum particle drift (See Appendix B). One gets the fit marked fit 1 putting the termination shock ~77AU. Fit 2 reduces diffusion parameters by 10%, and fit 3 assumes there is no contribution to the anomalous cosmic rays due to hydrogen.

Fig. 12



At first glance one might think that the fits do not match with the data points too well. This is because LS assumed an equilibrium flux, translating into a stationary termination shock. This is a good approximation during times in the solar cycle where the cosmic ray flux is relatively constant, and solar modulation is at a minimum, such as during the three-year period of 1975-1978 at solar minimum. Because the data used in this analysis by LS was during the period of 1991-1992 one can see that as the solar modulation sets in, the data points drop away from the fit. LS predict that after a return to equilibrium in the solar cycle, the data will "creep" upwards towards the fit. LS are currently analyzing data for the period of equilibrium flux covering 1996-1998, an equilibrium period. At the time of this paper, the solar cycle is currently approaching solar maximum and leaving the equilibrium period of 1996-1998.

B. Low Frequency Radio Emissions

Low frequency radio emissions detected in the inner heliosphere by plasma wave instruments aboard the Voyager 1 and Voyager 2 spacecraft were first used by Kurth, Gurnett, Scarf and Poynter (KGSP) as an alternative method to calculate a distance to the termination shock [34]. The Voyager spacecraft are on escape trajectories from the sun and are currently at 77 AU and 60.5 AU respectively, which is far beyond the orbit of Pluto at 30AU.

Charged particles such as electrons, respond to static and oscillatory electromagnetic fields within a plasma. In a fully thermalized, equilibrium state, electrons and other ions would oscillate about their equilibrium positions. If a plasma is perturbed, such as during the supersonic to subsonic transition that occurs by the solar wind at the termination shock, the perturbation to this equilibrium would displace the charged particles in such a way as to set up electromagnetic fields that would act as restoring forces to the displaced particles. A measure of the electric fields in perturbed plasma would show a strong resonance at a particular frequency known as the electron plasma frequency. The electron plasma frequency is proportional to the square root of the electron number density of the plasma.

$$F_p(\mathrm{Hz}) = ((4\pi q^2/m)*n)^{1/2} = 8980n^{1/2}$$

where m and q are the mass and charge of the electron and n is the electron number density in cm^{-3}. By measuring the characteristic frequencies of a plasma one may understand its properties such as the density and effects of magnetic fields which are carried in the plasma. Strong interactions, or instabilities, can occur between the plasma waves and the charged particles in the plasma. Electron plasma oscillations at the plasma frequency are frequently referred to as Langmuir waves. Plasma waves generally do not propagate very far in a plasma and are influenced enormously by the magnetized plasma when they do propagate. It is possible for the weakly propagating plasma waves to generate waves at frequencies much higher than the highest characteristic frequency of the plasma itself. These waves are electromagnetic and can propagate away from their source with very little interaction with the surrounding medium. These high frequency electromagnetic waves generated by the Langmuir waves are radio waves of plasma frequency F_p.

Voyager 1 and 2 possess identical plasma wave instruments that consist of four Faraday cup detectors, which measure ion and electron currents in the energy range 10-5959eV/Z. Three of the cups face into the solar wind, and the fourth cup is perpendicular to the solar wind direction and is used for detecting electrons. The currents obtained in the detectors are analyzed to find the speed, density, temperature, flux, and dynamic pressure of the solar wind. They also measure the electric field component of plasma and radio waves in the frequency range of 10Hz to 56 kHz.

In 1983 at heliocentric distances of 17AU for Voyager 1 and 12AU for Voyager 2, with a separation of ~10 AU between the two spacecraft, the Voyagers began detecting low intensity radio emissions at ~2-3 kHz. The emissions exhibited a low-frequency cutoff that was well above the local electron plasma frequency F_p, and the electron gyrofrequency in the solar wind. Also the frequency of the peak in the emission was nearly the same at both spacecraft despite their large separation, thus a local source for the plasma waves was ruled out, suggesting a freely propagating radio emission. Fig. 13 illustrates the relative power with respect to the observed frequency of the emissions.

Fig. 13

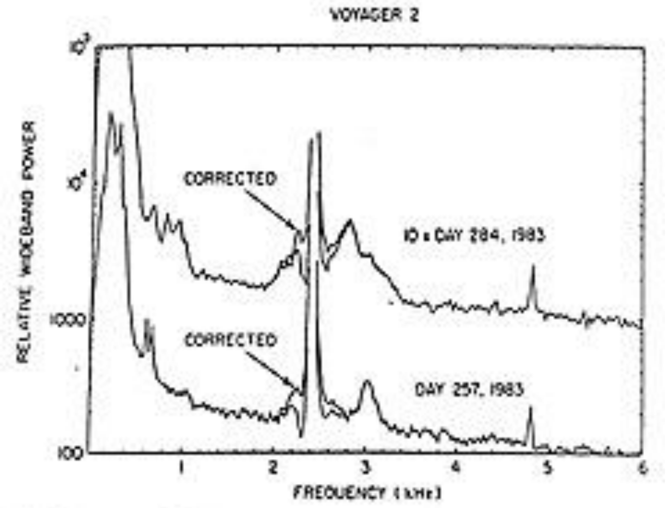
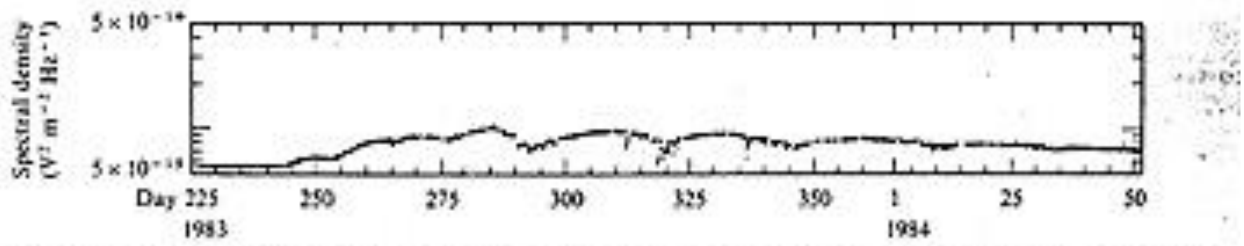


Fig. 3. Voyager 2 wideband spectra on days 257 and 283, 1983

In figure 13, the intense tones at frequencies below 1kHz are due to interference from the operation of the onboard tape recorder used to record the wideband data. The very narrow tones at 2.4 and 4.8 kHz are the first and second harmonics of the spacecraft power supply.

Unlike the 2kHz signal, (the 2kHz signal continued to be observed after the 3kHz emission disappeared in early 1984) the measured 3kHz signal was time dependent as shown in figure 14.

Fig. 14



Electric field spectral densities from the Voyager 1 3.11-kHz plasma wave receiver channel plotted as a function of time for slightly more than six months. Each point represents a 51.2-min average. The radio emission first appeared on day 243 1983.

After ruling out planetary sources, mainly due to the extremely low intensities of planetary radio signals, the termination shock was elected by KGSP to be the most likely source of the measured radio signals. If correct, these measurements would be the first direct contact with the termination shock. Shocks such as the termination shock have been theorized to produce weak narrow banded radio emissions at two times the plasma frequency, $2F_p$ due to plasma disturbances and mass enhancements at the shock [35-36]. The measured radio emissions characteristic smooth variance in time and bandwidth were also found by KGSP to be consistent with the $2F_p$ models.

Assuming that the termination shock is generating radio emissions at $2F_p$, one can make a crude estimate of its radial distance from the sun. Taking the $2F_p$ model to be valid, the plasma frequency at the shock must then be $F_p = 3\text{kHz}/2 = 1.5\text{kHz}$. If one can expect a maximum jump in the plasma density at the shock to be of a factor of 4, then the plasma density at the termination shock can be as low as $.008\text{cm}^{-3}$.

$$F_p(\text{Hz}) = ((4\pi q^2/m)*n)^{1/2} = 8980n^{1/2} = 1.5*10^3\text{Hz}$$
$$n = .03\text{cm}^{-3}\ ;\quad n/4 = .008\ \text{cm}^{-3}$$

This corresponds to a plasma frequency of about 780 Hz. Because the density of the solar wind falls off at $1/R^2$, (assuming constant solar wind speed) the plasma frequency varies as 1/R. Given that F_p was measured by the voyager instruments to be ~2kHz at 18 AU and drops to 780 Hz at the termination shock, one can solve for a radial distance to the shock that is ~46AU. This is obviously much smaller than the previous cosmic ray gradient measurements predict. One of the assumptions made by KGSP is that there are no variations in solar wind density as a function of time. This of course is known to be false. Also shocks are not well understood, and the 2Fp model most likely is in need of modification.

McNutt first suggested in 1988 that a high-speed solar wind stream or transient, could trigger a higher than normal intensity radio emission when it reached the termination shock [38]. This mechanism could be used to explain the time dependent 3 kHz radio emissions first reported by KGSP. The sun occasionally ejects an energetic burst of plasma called a coronal mass ejection, or CME. The ejection, (often associated with large solar flares) produces an interplanetary shock wave that propagates radially outward from the sun at velocities up to 1000 km/s. As the shock propagates outward the turbulent magnetic fields inhibit the entry of cosmic rays. This decrease in cosmic ray intensity is called a Forbush decrease and can be measured with spacebourne and earth based instruments. As a CME overtakes a spacecraft it can be identified by increased particle fluxes, and increased solar wind speed, followed by a Forbush decrease. A CME can contain an average of 10^{11} grams of coronal matter and pushes particles in its path radially outward. If this wall of dense solar wind plasma were to approach the termination shock, the density of the particles in the cavity would be enhanced. The trapped electrons bouncing back and forth in the cavity could resonate, and this would produce a radio emission and or Langmuir waves at F_p the plasma frequency.

McNutt found that two high-speed streams passed the Voyagers, and Pioneer 11 in 1983, just before the 3kHz radio emission was observed by the spacecraft. The solar wind speed was measured to be ~700 km/s by Pioneer 11, almost twice the normal solar wind speed. Peak solar wind speeds were measured by the Voyagers (hourly averages) to be 755km/s and 780 km/s on days 358 of 1982 and days 17 and 20 of 1983. McNutt conjectured that the lack of any strong 3kHz emissions from 1983 till the time of publication of his paper was due to the lack of any high-speed transients travelling past the Voyagers to the heliopause at that time. The lack of any measured high-speed streams by Pioneer 10 further supported McNutt's argument for a CME, or transient of solar origin.

McNutt was able to make inferences on the distance to the heliospheric termination shock by using time of flight arguments for CME's. Voyager II measured the stream structure when the spacecraft was between 10.95 AU and 11.09 AU on day 358 of 1982 and day 20 of 1983 respectively. The ~3kHz radio signal was first detected by Voyager I on day 242 in 1983 with a peak intensity recorded on day 285. By taking the time from the passage of the peaks in solar wind speed on day 358 of 1982 and day 20 of 1983 to the peak in emission, McNutt obtained transit times of the CME to the termination shock of 292 and 265 days. McNutt took the maximum distance to the termination shock to be given by the largest assumed transit speed of the CME. McNutt assumed the largest transit speed to be the peak stream speed measured at the Voyagers. The measured stream speeds were 755km/s and 780km/s. These values gave maximum distances to the termination shock of 138 AU and 130 AU respectively neglecting the light travel time of 1 day. For a minimum value, McNutt assumed that the transit speed for the disturbances must be at least greater than the supersonic non transient solar wind speed of 400 km/s. Using this speed in tandem with a 265 day travel time gave 72.3 AU as the limiting distance. McNutt's ranges of values for the termination shock's distance from the sun are much more in line with other models, than the KGSP model.

Steinolfson and Gurnett analyzed new radio emissions detected by the Voyagers in 1992 preceded by high-speed disturbances. These disturbances were thought to have originated at the sun during a period of intense solar activity [40]. Using a more sophisticated numerical simulation of CME propagation through the heliosphere their model predicted that the termination shock is located at 92 A.U.

Cairns, Kurth, and Gurnett, refined the KGSP 2Fp model in 1992 by suggesting that the radiation itself may undergo multiple frequency "up-shifts" by density enhancements whenever the local plasma frequency exceeds the radiation frequency [39]. This up-shifting mechanism allows the radio waves to propagate through the solar wind to much larger distances, putting the KGSP model in line with similar models. Considering that the Voyager I is now at ~77 AU and Voyager II is at 60.5 AU as of May 2000, and they still have not crossed the termination shock, any model requiring termination shock distances less than this value is certainly incorrect.

Another method of calculating the distance to the termination shock, which will not be explored in detail in this paper but deserves mention, involves measuring relativistic electron gradients. Relativistic electrons believed to be accelerated at the termination shock are measured by spacecraft, and identical calculations are made to those used by LS with the diffusion equation model.

VII. Review

The following table presents the different measuring techniques and their results for radial distances to the termination shock:

Year	Group	University	Calculated T.S.distance A.U.	Pressure balance	Galactic Cosmic Rays	Anomalous Component	Radio Waves
1955	Davis	Cal Tech	200	X			
1963	Parker	U of C	45-90	X			
1985	Randall VanAllen	U of Iowa	42-88		X		
1990-1999	Stone Cummings	Cal Tech	69-92 with 83.7 best fit			X	
1991	Lopate Simpson	U of C	70-90			X	
1984	Kurth Gurnett Scarf Poynter	U of Iowa	46				X
1988	McNutt	MIT	72.3-138				X
1995	Steinolfson and Gurnett	U of Iowa	92			X	

VIII. Discussion

A. Parker

All heliospheric models with respect to the termination shock, for the most part, are modifications upon the original model by Parker. Therefore, Parker's simplifying assumptions carry over into the other models. Also, unlike other theoretical frameworks, Parker's has withstood the test of multiple measurements to first order. No spacecraft has currently gone beyond 90 AU (Parker's maximum calculated distance to the termination shock) or passed through the termination shock. So Parker's assumptions in general have not been greatly undermined as of yet by direct measurement, the true test of any theory.

Parker's pressure balance argument assumes a perfectly spherical heliosphere with a stationary heliopause and termination shock. Parker assumes that the quiet time solar wind flows radially outward with constant a constant velocity of ~400 km/s and is an irrotatational flow, which implies that flow on each stream line ultimately comes to the same velocity and pressure at infinity. Parker assumes a smooth model of the solar wind with a uniform coronal expansion, and a uniform interstellar pressure.

We know that a perfectly spherical heliosphere is not correct, especially in the outer heliosphere. Since the termination shock is in the outer heliosphere, this aspect of the model could use some enhancement. Non-spherical heliospheric models have been examined by Fichtner, Fahr, and Sreenivasan [41]. Unfortunately, effects of a non-spherical heliosphere are observable only at a transition boundary, and no spacecraft has currently passed that boundary.

Parker's assumption of a stationary heliopause and termination shock is also incorrect, but is a good approximation over short time periods during solar minimum. Models taking into account the motion of the termination shock have been discussed in this paper.

The assumption of an irrotational solar wind flow of constant velocity seems to be true, especially close to the ecliptic out to distances of 60 AU - 77AU currently explored by the Voyager and Pioneer spacecraft.

Parker's assumptions of a smooth solar wind and a uniform coronal expansion have not been contested by any current theories. No measurements have been made to validate or contest uniform coronal expansion, so at the current time it can be assumed to be true. The Solar Probe space experiment will hopefully in the near future examine this topic experimentally.

B. Randall Van Allen

The Randall Van Allen model is more of a simple empirical fit to data than an actual model. RVA most importantly demonstrated that as one approaches the source of the galactic cosmic rays there is an increase in particle flux. RVA's only assumption was that the average radial gradients of cosmic rays were proportional to the logarithmic ratio of the particle fluxes at two spacecraft divided by their distance, with the phase of the solar activity cycle as the implicit variable. RVA's minimum distance of 42 AU during times of low solar modulation has been proven experimentally to be wrong though, since the Voyagers have been beyond this point for multiple solar cycles.

C. Stone and Cummings

Stone and Cummings show that the degree of tilt of the heliospheric neutral current sheet, which affects the access of the anomalous cosmic rays, results in a relationship between the current sheet tilt and the anomalous components intensity gradient. Their main assumption is that a radial profile of the tilt translates to a radial profile of the gradient. This assumption is reinforced by experimental data and physical arguments. Because their model reproduces observations made in the heliosphere it is better than many, and unlike the RVA model, that of Cummings and Stone has a strong physical basis, taking into account particle diffusion and drift models along with their fits to data measured in the heliosphere. Their prediction of 69 to 92 AU is also satisfying when compared with the ensemble of calculations and measurements reviewed in this paper.

D. Lopate and Simpson

The diffusion equation was first used as a heliospheric modulation model by Parker. All of the basic simplifying assumptions that Parker made are identical to those in the LS calculations. When one examines figure 12, it becomes apparent that the stationary termination shock paradigm is limited to times of low solar modulation. Alternatively, the fits during solar minimum are quite good, and the chances are that they will improve once more with analysis of the 1996-1998 “quiet” solar wind data. Their calculated shock distances also are in a very reasonable range of 70AU- 90 AU.

E. Kurth, Gurnett, Scarf, and Poynter

Although it is apparent that the KGSP predictions were incorrect, development of their basic assumptions seems promising for the future. Also the assertion that the radio waves measured by the Voyager spacecraft originate at the termination shock is almost certainly true. The Cairns, Kurth and Gurnett frequency enhancement modifications seem to suggest more acceptable results can be had with the basic $2F_p$ model. Although one must be careful not to associate “reasonable” values with physical truth too quickly.

F. McNutt

McNutt’s observations of high-speed solar wind streams as a precursor to intense radio wave emissions have now been reproduced on multiple occasions. As we enter into solar maximum, and the Voyagers continue to approach the termination shock, it is very likely that more of these disturbances will be measured. McNutt’s simple time of flight calculations gave reasonable termination shock distances, and a better understanding of CME propagation coupled with more sophisticated models suggests that this method may be a very sound and straightforward way to approximate the termination shock’s distance.

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I would like to thank my thesis advisor, Dr. John Simpson, for suggesting such an interesting and exciting topic, and for sharing his vast knowledge on cosmic rays and the heliosphere. Many useful discussions with Dr. Clifford Lopate on shock acceleration, heliospheric dynamics, the anomalous component, and all things astrophysical, are also greatly acknowledged.

References

[1] Parker, E. N.: *Interplanetary Dynamical Processes* Interscience, New York, 1963 **[2]** L.A. Fisk, B. Koslovsky, R.Ramaty, Astrophys. J., Vol 190: L35-L37, 1974 **[3]** Simpson J.A. Proc. Int. Cosmic Ray Conf., Moscow, 9, 84 (1987) **[4]** Axford W.I. Physics of the Outer Heliosphere: Proceedings of the 1st COSPAR Colloquium Ed. S Grzedzielski and D.E. Page. p.7, 1990 **[5]** P. Meyer, E. N. Parker, and J.A. Simpson, Physical Review Vol. 104: 768-783, (1956) **[6]** L.E. Davis, Jr., Phys. Rev., 100, 1440 (1955) **[7]** L. Biermann Z. Astrophysik 29, 274 (1951) **[8]** Axford, W.I. 1972, in *Solar Wind*, ed. C.P. Sonett, P.J. Coleman, and J.M. Wilcox, NASA SP-308 (Washinton, D.C.: Nationa Aeronautics and Space Administration), p. 609 **[9]** Parker, E. N., Astrophys. J., Vol. 134, 1961, p.20. **[10]** T. Clauser, 4th Symp. Cosmical Gas Dynamics, Varenna (1960) **[10]** Weymann, R., Astrophys. J., Vol. 132, 1960, p.380 **[11]** Jokipii, J.R., Astrophys.J.,152,799,1968. **[12]** M.Garcia-Munoz, G.M. Mason, and J.A.Simpson, Astrophy. J. (Letters), 182, L81,(1973). **[13]** D.Hovestadt, O. Vollmer, G.Gloeckler, and C.Y.Fan, Phys. Rev. Letters, 31, 650, (1973). **[14]**F.M Mcdonald, B.J.Teegarden, J.H.Trainor, and W.R.Webber, Astrophys. J. (Letters), 187, L105, (1974). **[15]** M.E. Pesses, J.R. Jokipii, and D. Eichler, Astrophys. J. (Letters), 246, L85, (1981) **[16]** Gloeckler, G., and Jokipii, J.R., Astrophys. J. (Letters), 148, L41, (1967). **[17]** Hovestadt, D.B., Klecker, B., Gloeckler, G., Ipavich, F.M., Fan, C. Y., and Fisk, L.A. Proc. 15th Internat. Conf. Cosmic Rays (Kyoto), 3, 255, (1979). **[18]** McKibben, R.B., Pyle, K. R., and Simpson, J. A., Astrophys. J. (Letters), 227, L147 (1979). **[19a]** Randall, B.A., and Van Allen, J.A. Geophys. Res. Lett., 13, 628, (1986). **[19b]** Van Allen, J.A., Randall, B.A., J. Geophys. Res., 90, 1399, (1985) **[20]** Gleeson, L.J., and W.I. Axford, Astrophys. J. 154, 1011, (1968). **[21]** Meyer, P., Ann. Rev Astron. Astrophys., 7, 1, (1969) **[22]** Ryan, M.J., Ormes, J.F., Balasubrahmanyan, Phys. Rev. Lett., 28, 985, (1972). BB **[23]** Fulks, G.J., J.Geophys. Res., 80, 1701, (1975). **[24]** Cummings, A.C, Stone, E.C., and Webber, W.R., Proc. 21st Internat. Cosmic Ray Conf., 6, 190, (1990). **[25]** Stone, E.C., Cummings, A.C., and Webber, W.R., Proc. 22nd Internat. Cosmic Ray Conf., 3, 386, (1991). **[26]** Cummings, A.C., Stone, E.C., J. Geophys. Res., 98, 15165, (1993). **[27]** Cummings, A.C., Stone, E.C. J. Geophys. Res. 99, 11547, (1994). **[28]** Stone, E.C., Cummings, A.C., Webber, W.R., Proc 24th Internat. Cosmic Ray Conf., , 792, (1995). **[29]** Stone, E.C., Cummings, A.C. Proc. 26th Internat. Cosmic Ray Conf., SH 4.2.03, 500, (1999). **[30]** Jokipii, J.R., Levy, E.H., Hubbard, W.B. Astrophys. J., 213, 861, (1977). **[31]** Lopate, C, Simpson, J.A., J. Geophys. Res., 96, 15, 877, (1991). **[32]** LeRoux, J.A., Potgieter, M.S. Astrophys. J., 390, 661, (1992). **[33]** Lopate, C., Simpson, J.A. Proc. 23rd Cosmic Ray Conf., 3, 415, (1993) **[34]** Kurth, W.S., Gurnett, D.A., Scarf, F.L., Poynter, R.L., Nature, 312, 27, (1984). **[35]** Fung, S.F., Papadopoulos, K., Wu, C.S., J. Geophys. Res., 87, 8077, (1982). **[36]** Cairns, I.H., Melrose, D.B., J. Geophys. Res., A7, 6637, (1985) **[37]** Foreman, M. A., 1985, in *Collisionless Shocks in the Heliosphere: A Tutorial Reveiew*, ed. Robert G. Stone, and Bruce T. Tsurutani, 91, American Geophysical Union, Washington D.C. **[38]** McNutt, R.L., Geophys. Res. Lett., 15, 1307, (1988). **[39]** Cairns, I.H., Kurth, W.S., Gurnett, D.A., J. Geophys Res., 97, 6245, (1992) **[40]** Steinolfson, R.S., Gurnett, D.A., Geophys. Res. Lett., 22, 651, (1995)

Appendix A.

The demodulation model of Gleeson and Axford assumes that the effective change in energy of a charged particle moving from a source point P on the boundary along a dynamical trajectory regardless of its path to an interior point Q with a kinetic energy T′, must have had a kinetic energy (T′ + Zeϕ) at P [20]. RVA terms the unidirectional differential energy intensity $j'_Q(T')$ at Q along a dynamical trajectory leading from P to Q to be related to that at P in a "quasi-steady state" by:

(6) $j'_Q(T')/\ (T'(\ T'+2Amc^2)) = j'_p(T' + Ze\phi)/((T' + Ze\phi)(T' + Ze\phi + 2Amc^2))$

where A is the atomic mass number of a particular species of energetic ion, Ze is its positive charge, mc^2 is the rest energy of the ion at the observing point Q and (T′ + Zeϕ) is its kinetic energy at the boundary, where ϕ is positive for ions and negative for electrons.

Next RVA denoted the energy per nucleon as T, such that T′ = AT so (6) becomes:

(7) $j_Q(T)/\ (T(\ T+2mc^2)) = j_p(T + Ze\phi/A)/((T + Ze\phi/A)(T + Ze\phi/A + 2mc^2))$

RVA used only observed data for protons and alpha particles giving (Z/A = 1) and (Z/A = .5) respectively.

RVA next made the assumption that the high energy (T > 30,000 MeV/nucleon) portions of the differential spectra of the primary protons and alpha particles at 1A.U. can be well represented by:

(8) $j(T) = K/(T+mc^2)^\gamma$

where γ is the spectral index γ = 2.7 as shown in Meyer, 1969, Ryan et al., 1972, and Fulks, 1975 with the added assumption that (8) is valid down to T = 80 MeV/nucleon in the nearby interstellar medium [21, 22, 23].

Next RVA combined equations (7) and (8) substituting Φ for Zeϕ/A giving two working formula for the unidirectional spectral intensity $j_{p\alpha}(T)$ of alpha particles or protons at an observing point Q where T is the kinetic energy per nucleon.

(9) $j_{p\alpha}(T) = (T(\ T+2mc^2)/((\ T +\Phi_{p,\alpha}\ )(\ T +\Phi_{p,\alpha} + 2mc^2))\ x\ (K_{\ p,\alpha})/((\ T +\Phi_{p,\alpha} + mc^2)^{\gamma p\alpha}$

Next RVA made least square fits to available 1 A.U. spectral data for protons and alpha particles in the energy range 10^2 MeV/nucleon < T < 10^6 MeV/nucleon using a logarithmic version of equation (9) in order to calculate values for K_p, γ_p, and Φ_p, along with K_α, γ_α, Φ_α where $\Phi_{p,\alpha}$ is a function of epoch taking $\Phi_\alpha = \Phi_p/2$. The values of the four time independent parameters being: $K_p = 1.99*10^5$, $\gamma_p = 2.84$, $K_\alpha = 4.27*10^4$, and $\gamma_\alpha = 2.84$ with j(T) measured in $(cm^2\ sec\ sr\ MeV/n)^{-1}$. Where the values of $\Phi_\alpha = \Phi_p = 0$ in interstellar space returns the interstellar counting rate of 1.83 counts/sec.

Appendix B.

The diffusion equation:

$$\partial U/\partial T = \nabla \bullet (\kappa_s \bullet \nabla U) - (V_{sw}+V_d) \bullet \nabla U + ((1/3)(\nabla \bullet V_{sw})(d(\alpha T U)/dT))$$

On the right side of the equation:

Term 1 accounts for inward diffusion of the accelerated anomalous component from the termination shock.

Term 2 accounts for outward convection and particle drift.

Term 3 accounts for adiabatic deceleration of particles in the heliosphere.

- U(R,E) ; energy spectrum of the anomalous component
- V_{sw} ; vector solar wind velocity
- $V_d = -(\nabla \bullet \kappa_a)$; velocity resulting from particle gradient and curvature drifts in the interplanetary magnetic fields
- T(R,E) ; The particle kinetic energy
- $\alpha= (T+2m)/(T+m)$; where m is the particles rest mass
- $\kappa^{(s)}(R,E)$; symmetric part of the diffusion tensor
- $\kappa^{(a)}(R,E)$; asymmetric part of the diffusion tensor

The diffusion tensor $\kappa_d = \kappa^{(s)} + \kappa^{(a)}$

$$K^s = \begin{bmatrix} Kx & 0 & 0 \\ 0 & Ky & 0 \\ 0 & 0 & Kz \end{bmatrix} \qquad K^a = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & Kt \\ 0 & -Kt & 0 \end{bmatrix}$$

- $\kappa_t = -C\beta P/3B$; diffusion transverse to the electromagnetic field B, β is the particles velocity, P is rigidity, and C = 1 at maximum drift, and C = 0 for no drift
- $\kappa_x= \kappa_o \beta P^{\alpha}$; diffusion parallel to the B field
- $\kappa_y = \kappa_z\ .01\kappa_x$ diffusion perpendicular to the B field

with α and κ_o as:

$\kappa_o=7.5*10^{21}\ \mathrm{cm}^2/\mathrm{s}$ $\quad \alpha = 0$ for $P < .3\mathrm{GV}$

$\kappa_o=25.0*10^{21}\ \mathrm{cm}^2/\mathrm{s}$ $\quad \alpha = 1$ for $P \geq .3\mathrm{GV}$

LS calculated values of R_o between 70 and 90 AU for various values of $\kappa^{(s)}$, and $\kappa^{(a)}$, by varying C and κ_o. Figure 12 shows fits for the different values of κ_o shown above, with C =1. One gets the fit marked fit 1 by putting the termination shock ~77AU. Fit 2 reduces κ_o by 10%, and fit 3 assumes there is no contribution to the anomalous cosmic rays due to hydrogen.